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Appendix: Basic Terminology

The first modern English dictionary came out in 1755. Before that, "dictionaries" were merely just a listing of unusual words, with perhaps a synonym or two next to each word. Once you have a dictionary, you can imagine keeping it up to date would require adding new words you come across. On the other hand, it would be incredibly difficult to write one from scratch. But Samuel Johnson was able to write one single handedly with 42,773 words, after seven years of work. It was a meticulous work, for example the word "take" had 134 definitions!

In this section, we'll do something much more modest; build a small vocabulary of a number of elementary, but important concepts, which we'll use repeatedly throughout the book. Once we have a working list, it'll be far easier to update our list whenever we run into a new term.

What is $3+2$?	$3+2=5$.
Is $3+2$ a number?	Yes, 5 is a number.
What is 4 + elephant?	We can't do such an addition, so the sum is undefined.
Is "elephant" a number?	No, because we cannot do arithmetic with "elephant".
Is $\spadesuit$ a number?	Maybe, if we defined arithmetic with $\spadesuit$. Otherwise, no.
If $\spadesuit+3=4$, what is $\spadesuit$?	Since $4-3=1$, we have $\spadesuit=1$.
What is $1+1+1+\cdots$?	Adding 1 infinitely many times gives **infinity**.
If ∞ denotes infinity, what is $\infty+1$?	The sum evaluates to $\infty+1=\infty$.
What is $3+\infty-\infty$?	Undefined; there is no way to resolve this expression.
Is ∞ a number?	No, we can't do some of the usual arithmetic with ∞.

A **set** is any collection of objects, written by enclosing its objects (called **elements**) inside { }.

Is $\{1, 2+3\}$ a set?

Yes, $\{1, 2+3\}$ is a set with two numbers 1 and 5.

Is {1, {elephant, cat}} a set?

Yes, notice one of its elements is a set of two animals.

The set of numbers we use to count: $\{0, 1, 2, 3, \dots\}$, is called the (set of) **natural numbers**.

Is 6 a natural number?

Yes, because it is a number we use to count items.

Is $\frac{1}{2}$ a natural number?

No, $\frac{1}{2}$ is not a number we use to count whole items.

Is -2 a natural number?

No, -2 is not a number we use to count items.

When an object a belongs to a set S, we write $a \in S$. For example, $2 \in \{0, 1, 2, 3, 4, 5, \dots\}$.

Is 5 billion $\in \{0, 1, 2, 3, 4, 5, \dots\}$?

Yes, because 5 billion is a counting number.

We denote the set of natural numbers with the symbol $\mathbb{N}$.

Is $\infty \in \mathbb{N}$?

No, because ∞ is not a number; ∞ is a symbol.

When an object a *does not* belong to a set S, we write $a \notin S$. For example, $\infty \notin \mathbb{N}$.

The **integers** $\mathbb{Z}$ is the set of negative and positive counting numbers: $\{\dots, -2, -1, 0, 1, 2, \dots\}$.

An **empty set** has no elements. We denote an empty set by the symbol $\emptyset$ or the symbol { }.

Is $\{\emptyset\}$ an empty set?

No, it is **nonempty**, as it contains a set as its element.

A set has no notion of ordering. Thus $\{1, 2, 3\}$ is the same set as $\{2, 3, 1\}$ and $\{3, 1, 2\}$.

Is {pear, fig} the same as {fig, pear}?

Certainly.

A set has no notion of element multiplicity. This means $\{1, 2, 3, 3, 3\}$ is the same set as $\{2, 3, 1\}$.

Is {1, {1}, cat} the same as {1, cat}?

No.

A **list** is an ordered sequence of objects (called elements), separated by | and enclosed inside [ ].

Is [1|cat|3] a list?

Yes, its elements are separated by | and enclosed in [ ].

Is $\{1, 2, 1\}$ a list?

No, because the objects are not enclosed in [ ].

Is [1|{1, 3}|2] a list?

Yes, it is a list with two numbers and one set.

<table>
<tr><td colspan="2">Sets and lists are examples of abstract data types which support various useful operations.</td></tr>
<tr><td colspan="2">The number of objects in a list is called length. We write that list L has length: $\text{len}(L)$.</td></tr>
<tr><td>What is the length of [1|2]?</td><td>The length of list [1|2] is 2.</td></tr>
<tr><td>What is len([1|cat|2])?</td><td>[1|cat|2] has two numbers and one animal, so 3.</td></tr>
<tr><td>What is len([1|$\{1,3\}$|2])?</td><td>[1|$\{1,3\}$|2] has two numbers and one set, so 3.</td></tr>
<tr><td colspan="2">The position of an object in an list is called index. We start the position from 0.</td></tr>
<tr><td>What is the index of 5 in [5|2|3]?</td><td>5 appears before any element in the list, so 0.</td></tr>
<tr><td>What is the index of 2 in [5|2|3]?</td><td>2 appears in position 1 in the list, so 1.</td></tr>
<tr><td>What is the index of 3 in [5|2|3]?</td><td>3 appears in position 2 in the list, so 2.</td></tr>
<tr><td>What is the index of 2 in $\{1,2\}$?</td><td>Not applicable; sets are unordered, so indexes are undefined.</td></tr>
<tr><td>Is [1|2] the same as [1|2|2]?</td><td>No, the lengths of the two lists are not the same.</td></tr>
<tr><td>Is [1|2] the same as [3|2]?</td><td>No the element in index 0 of each list are not the same.</td></tr>
<tr><td colspan="2">We see that unlike a set, in a list, the order of each element and number of elements matters.</td></tr>
<tr><td colspan="2">We use the symbol ‘=’ to indicate two sets (lists) are equal; if not, we use the symbol ‘$\neq$’.</td></tr>
<tr><td>Is $\{1,2,1,1,1\} = \{2,1\}$?</td><td>Yes, this is true.</td></tr>
<tr><td>Is [1|2] = [3|2]?</td><td>No, this is false; we write [1|2] $\neq$ [3|2].</td></tr>
<tr><td colspan="2">A twople is a list of two elements, written with parenthesis and a bar. For instance, (3|5).</td></tr>
<tr><td>Is [pear|5] a twople?</td><td>This is a list of two elements, but we agree to write (pear|5).</td></tr>
<tr><td>Is (pear|5|5) a twople?</td><td>No, because there are three items.</td></tr>
<tr><td>Is ($\{\text{pear}, 5\}$|3) a twople?</td><td>Yes, it contains two objects, a set $\{\text{pear}, 5\}$ and a number.</td></tr>
<tr><td>Is ([pear|5|2]|3) a twople?</td><td>Yes, it contains two objects, a list [pear|5|2] and a number.</td></tr>
<tr><td>Is (2|3) = (3|2)?</td><td>No, order matters for twoples.</td></tr>
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